

A TECHNICAL AND ECONOMETRIC STUDY
OF PORK BELLY STORAGE

by

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THESIS

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ABSTRACT

Various models relating to the storage of pork bellies are studied. An analysis of covariance is used to determine if specific regional weekly storage totals can be used to predict national monthly storage totals. A technical forecast model in the form of a Fourier series is presented, derived from past national storage data. Finally, an econometric model relating pork belly demand, change in storage supply, hog slaughter, and total pork belly supply is formulated and fitted to the data base.

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I. INTRODUCTION

The supply-demand relationships of commodities are oftentimes complex and consequently seldom well understood. The entire set of particulars governing each commodity is large and may defy specific categorization. Modeling techniques encompassing the commodity arena may take a variety of forms. Comparison of competing models is of interest as is the quantification of the results of individual models in terms of generally accepted criteria.

Prior to an in-depth analysis of a particular commodity, one may observe various phenomena associated with that commodity. One such phenomenon is the seasonality or cyclical patterns that may be exhibited by a commodity with regard to both supply and demand. This may have enormous effects on price, as viewed by classical economics, especially when supply and demand are not in phase. One way of reducing excessive price fluctuations is to store the commodity in periods of relatively high supply for consumption later in periods of relatively high demand. In this regard, the futures' market functions to provide a forward pricing mechanism enabling the various segments of an industry to minimize their risks by hedging [Ref. 9]. Although the existence of storage facilities may add to the framework under which a commodity study must be structured, it does not necessarily confound it. Specifically, storage facilities, by the very nature of their operations, may possess cyclical patterns

having sufficient definition to aid in the study of a commodity.

This paper is concerned with characterizing storage within the pork belly market, a commodity possessing the aforementioned traits. In addition, this market enjoys virtual freedom from foreign competition and constraints imposed by government price supports, two factors that further add to the complexity of a study. The extensive data published [Ref. 6] provides a substantial base from which to postulate a variety of technical and fundamental models of the pork belly market. However, the form in which the data is presented influences the method of modeling and its limitations constrain the model's detail. Naturally the success of a model depends on how judicious is the choice of assumptions on which the model is structured.

Two technical models are proposed to predict pork belly storage movements based on historical records alone. Standard least squares theory was applied to estimate the necessary parameters in the relations. In addition, a fundamental model is proposed that relies on generally accepted econometric methods. This model was structured in terms of variables thought to be pertinent to the pork belly market and their economic import is discussed with regard to the analysis. A system of five equations serve to describe the model. Because of overidentification, the method of two-stage least squares was used to estimate the structural coefficients involved. The object of this model is not only

to predict storage movements but also to attempt to identify some of the economic forces influencing the pork belly market.

Section II contains some background information and specific details of the hog-pork belly interface plus the results of a literature search that were useful in this study. The technical models are presented in Sections III and IV. The fundamental model with its five equations is discussed in Section V, followed by a summary of results in Section VI.

II. NATURE OF THE PROBLEM

A. BACKGROUND

As hogs are slaughtered, the underside portion of the carcass, called the pork belly and more commonly known in its processed form as bacon, is removed. Each hog yields two pork bellies each ranging in weight from 8 to 40 pounds depending on the size and age of the hog, the average estimated to be 12 to 13 pounds [Ref 2]. Typical choice, bacon-producing bellies range between 10 and 16 pounds and are taken from barrows and gilts in the 190 to 240 pound weight class. After initial processing, the pork belly is either sliced for bacon immediately or packaged and stored frozen to be sliced into bacon at a later date. This storage process helps reduce price fluctuations by augmenting fresh supply during periods of high demand for bacon. The freezer facilities involved attempt to take advantage of the supply-demand relation in maximizing their profits over the long term.

Hog slaughter in the nation is subject to seasonal fluctuations due to the fact that the majority of hogs are slaughtered in the fall and early spring. Since bacon consumption reaches a peak during the summer months, pork bellies are stored during the peak slaughter periods to be delivered during the periods of peak demand. Historically there has been net into-storage movements of pork bellies for the eight months of October through May and net out-of-storage movements for the four months June through September.

Another factor contributing to the storage cycle just mentioned, is the Commodity Exchange requirement that pork bellies stored prior to 1 December of one year are not deliverable on a futures' contract the following year [Ref. 6]. This rule in effect insures a complete turn-over of storage stocks yearly.

The farmer attempts to adjust to the economic forces of the market by varying his pig crop from farrowing to farrowing, year to year. In so doing, the subsequent hog slaughter not only varies over the course of the year as described, but the total yearly slaughter also varies as well. As a consequence of this, pork belly storage movements have also varied yearly as freezer facilities adjusted to the total pork belly supply. Total storage at any one time is really a secondary supply awaiting later relative demand increases. The magnitude of the total storage, in light of anticipated future supply and demand, must have obvious effects on not only the cash price for pork bellies but the futures' prices as well. Thus it becomes advantageous to one dealing in this commodity to be able to predict with some degree of certainty, storage figures in advance. In so doing, a freezer facility for example, might resort to hedging or take advantage of any disparity between existing prices and estimated prices derived from the forecasted supply.

B. BASIC ASSUMPTIONS

Reference 6 provides data on total hog slaughter (in thousands of head) by day, week, and month for federally

inspected slaughter houses in the nation. Barrow and gilt slaughter figures are given by the month only. Weekly bacon slice (in thousands of pounds, including both fresh and frozen pork bellies), weekly total pork belly storage in Chicago and Outside (in thousands of pounds), and monthly national pork belly storage totals (in thousands of pounds) are also provided. Although the average live and dressed weight for all hogs slaughtered is given weekly, there is no breakdown as to actual weight distribution. Two other important missing variables are the total number of pork bellies taken and in storage along with their corresponding weight distribution. Although pork bellies are required to be stored within fifteen days of slaughter [Ref. 6], no data is given for their actual time in transit from slaughter house to freezer warehouse.

Based on an initial assumption that supply equals demand at any given time, hog slaughter was considered as the sole determinant of the fresh supply of pork bellies. Total bacon slice was considered as part of the demand since this figure should reflect immediate consumer desires. On the other hand, net storage movements, because of their cyclical nature, were considered as part of the demand during in-movements and part of supply during out-movements. Because of the way the data are presented, transitions from one state to another were considered instantaneous (e.g., bacon slice and storage change figures for an in-month were a result of the fresh pork bellies derived from hog slaughter

for that month). The following equations represent these assumptions:

$$\alpha_t Q_t = BS_t + \Delta S; \text{ for into-storage movements during} \\ \text{month } t \quad (1.0)$$

$$BS_t = \alpha_t Q_t - \Delta S; \text{ for out-of-storage movements during} \\ \text{month } t \quad (1.1)$$

where

Q_t = number of barrows and gilts slaughtered in
month t ;

BS_t = monthly bacon slice in month t , interpolated
from weekly totals (in pounds);

$\Delta S = S_t - S_{t-1}$, net change in pork belly storage (in
pounds) in month t ;

α_t = pounds of pork bellies per hog slaughtered in
month t .

Table I contains computed values of α_t using data from the period January 1967 through December 1969. For the sake of clarity, α_{ij} was substituted for α_t , where $j=67, 68$, and 69 for years, and $i=1,2,\dots,12$ for months. It might seem reasonable to suppose that α_t would be constant over time (e.g., a 220 pound hog would yield two bellies whose weight was distributed $N(25, \sigma^2)$ for all t). However, as can be seen from Table I, α_t was not constant over time, although the values obtained appeared to be fairly consistent for the same month from year to year. The values obtained seemed to be in marked disagreement with Stoken's estimation of the average weight of pork bellies per hog, which was 25 pounds

[Ref. 2]. This discrepancy could have resulted from incomplete data (i.e., that the actual total weight of pork bellies taken from the hog slaughter was not reported), or the possibility that pork bellies were not taken from every hog, or both. Although the reason for the discrepancy is still unresolved, its effect can be circumvented by considering the apparent consistency of α_t from year to year (i.e., $\alpha_{i1}=\alpha_{i2}=\alpha_{i3}$). It can be shown that the correlation coefficient obtained from observations of two random variables functionally related, is unchanged if the observations of one of these random variables is in error by a constant amount. Therefore it was assumed that any omissions in the data was constant and did not affect the correlation between hog slaughter and pork bellies obtained. However this did not explain the variability of α_t from month to month over the year.

Two plausible but unsubstantiated explanations as to the cause of the variability in α_t over the year are firstly, that an animal subjected to varying climatic conditions would adapt physiologically to those conditions. A hog would produce a different consistency of fat in the abdominal area during the winter than in summer. Secondly, it is customary to slaughter breed stock during the summer months and the resulting bellies might be unsuitable for bacon production and thus go untallied.

C. LITERATURE SEARCH

Various previously proposed fundamental models were studied and evaluated. One related model proposed by Wold

[Ref. 8], was an attempt to identify aspects of the hog market on a yearly basis. This formulation, although yielding acceptable results for its purpose, failed to identify the possible short term influences in the hog market which in turn might be useful in studying the pork belly market. Another related study by Leuthold, et al, [Ref. 9], developed several schemes to predict daily hog prices and quantities, using various forecasting techniques. The time frame involved in this model was considered as being too short to allow identification of the major economic forces in the market. Any information gleaned from these methods were thought to be of marginal use in studying the cyclical nature of the hog and pork belly storage markets.

Because of the cyclical nature of the hog supply, a spectral analysis was performed relating hog supply with hog price, hog-corn price ratio, and pork belly price. Although some evidence of correlation between these variables was noticed for different time lags, no meaningful interpretation could be attached. Any attempts to smooth out random disturbance effects also affected the meaningful portions of the cross spectra. Any further analysis along this vain would require considerably more experience in the use of smoothing techniques as well as a more complete and basic understanding of the market forces involved.

A pamphlet by Stoken [Ref. 2] although informative, was written mainly for people wishing to speculate in the pork belly futures' market. This paper did however, identify various pertinent relations germane to the pork belly market

and in this respect was most helpful in the conduct of this study.

The fundamental model actually used was a result of a similar study involving cattle and hog supply-demand relationships conducted by Hayenga and Hacklander [Ref. 1]. Their method was thought to give more meaningful results in light of the main objectives of this study.

The periodicity indicative of the pork belly storage seasons, appeared to be unique in comparison to other commodity studies. Thus the two technical models proposed resulted in the use of rather basic techniques commonly associated with similar phenomena in other unrelated studies.

III. TECHNICAL MODEL I

Statistics are published weekly on the net pork belly storage movements for approved Mercantile Exchange warehouses in Chicago and Outside (i.e., area immediately surrounding Chicago). National figures for net storage movements are published by the month. Assuming that inferences might be made about pork belly prices both fresh and frozen, from total national storage figures, it might be advantageous to be able to predict the monthly national totals from weekly regional totals. Figure 1 is a plot of national storage versus Chicago and Outside for the periods indicated. From this graph, one might hypothesize that each complete line segment has the same slope (i.e., that the proportion of storage totals for the nation to Chicago and Outside is a constant for both in and out storage movements). To test this hypothesis, the following analysis of covariance was conducted using data obtained for the three year period June 1967 to May 1970 inclusive:

$$y_{ijk} = \mu_{ij} - \beta_{ij}(x_{ijk} - \bar{x}_{ij}) + e_{ijk} \quad (2.0)$$

where

$i = 1, 2, 3$ years

$j = \begin{array}{l} 1 \text{ during in-movements} \\ 2 \text{ during out-movements} \end{array}$

$k = 1, 2, \dots, n_j$ months; $n_1=8, n_2=4$

and

y_{ijk} = national storage total

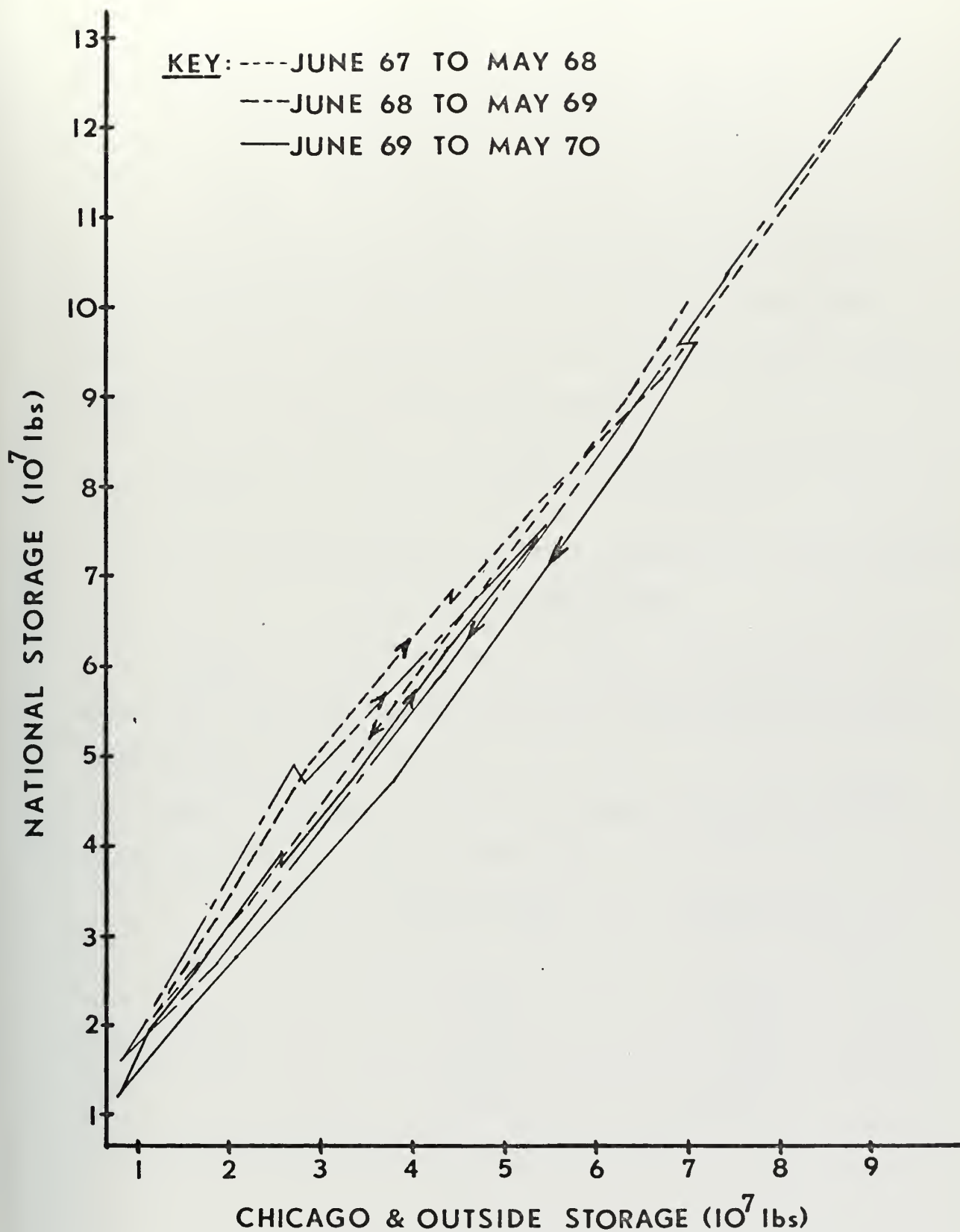


figure 1

x_{ijk} = Chicago and Outside storage total

β_{ij} = regression coefficient (unknown)

μ_{ij} = national storage mean (unknown)

e_{ijk} = error term; distributed $N(0, \sigma^2)$.

Since it was to be determined if the total storage in Chicago and Outside paralleled the national storage for both in and out movements from year to year, the following hypothesis testing problem was proposed:

$H_0: \beta_{ij} = \beta, \forall i, j$; for the null hypothesis,

$H_1: \beta_{ij} \neq \beta, \forall i, j$; for the alternative at the 5% level of significance.

Decomposition of the sum of squares of equation (2.0) will yield the operational table for the analysis of covariance listed in Table II.

Results obtained for the estimates of β_{ij} are tabled below where year 1 begins June 1967:

	$\hat{\beta}_{ij}$	Out-Movement	In-Movement
	1	1.324	1.255
Years	2	1.335	1.216
	3	1.273	1.318

The estimate for β was found to be:

$$\hat{\beta} = 1.275.$$

The unbiased estimate of the standard deviation of residuals σ was found to be:

$$\hat{\sigma} = 1938 \times 10^3 \text{ pounds.}$$

Under the null hypothesis the test statistic $MS_{\text{slope}}/MS_{\text{within}}$ (Table II) is F distributed with the appropriate degrees of freedom. In this case the test statistic was found to be 1.67, which was less than the tabled value for the F distribution: $F_{(5,24)}=2.62$, at the .05 level of significance. Thus the null hypothesis was accepted and weekly storage totals at Chicago and Outside can be used in conjunction with $\hat{\beta}$ to predict the national monthly storage totals.

The $(100-\alpha)\%$ confidence limits for β and σ were also computed and found to be:

$$1.238 \leq \beta \leq 1.312$$

$$1513 \times 10^3 \leq \sigma \leq 2697 \times 10^3; \text{ where } \alpha=.05$$

The theory and necessary procedures required to determine the preceding estimators and confidence limits may be found in Ref. 3.

IV. TECHNICAL MODEL II

The seasonal cycle indicative of pork belly storage may be seen in Figure 2, part of which is a scatter diagram of national storage versus time. Another feature of Figure 2, in addition to the seasonal cycle, is the apparent four year period between peaks of maximum yearly storages as noticed by Stoken [Ref. 2]. To explain these effects mathematically, the following Fourier series was developed in an effort to fit the scatter diagram of observations covering 12 years, from November 1957 to October 1969 inclusive:

$$Y(t) = b_1 + b_2 \cos \frac{2\pi X(t)}{16} + b_3 \sin \frac{2\pi X(t)}{16} \\ + b_4 \cos \frac{2\pi X(t)}{64} + b_5 \sin \frac{2\pi X(t)}{64} + e(t) \quad (3.0)$$

where

$Y(t)$ = storage total (in 10^3 lbs) at time t ; $t=1,2,\dots,144$

$X(t)$ = a transformation on t (see below)

b_i = regression coefficient (unknown); $i=1,\dots,5$

$e(t)$ = error term.

To account for the fact that the observed data does not follow a true sinusoidal function, since there are eight months from trough to peak and only four months from peak to trough, the following compensating relations were developed:

$$X(t) = \begin{cases} 16s + r & ; 0 \leq r \leq 8 \\ 16s + 8 + 2(r-8) & ; 8 < r \leq 11 \end{cases}$$

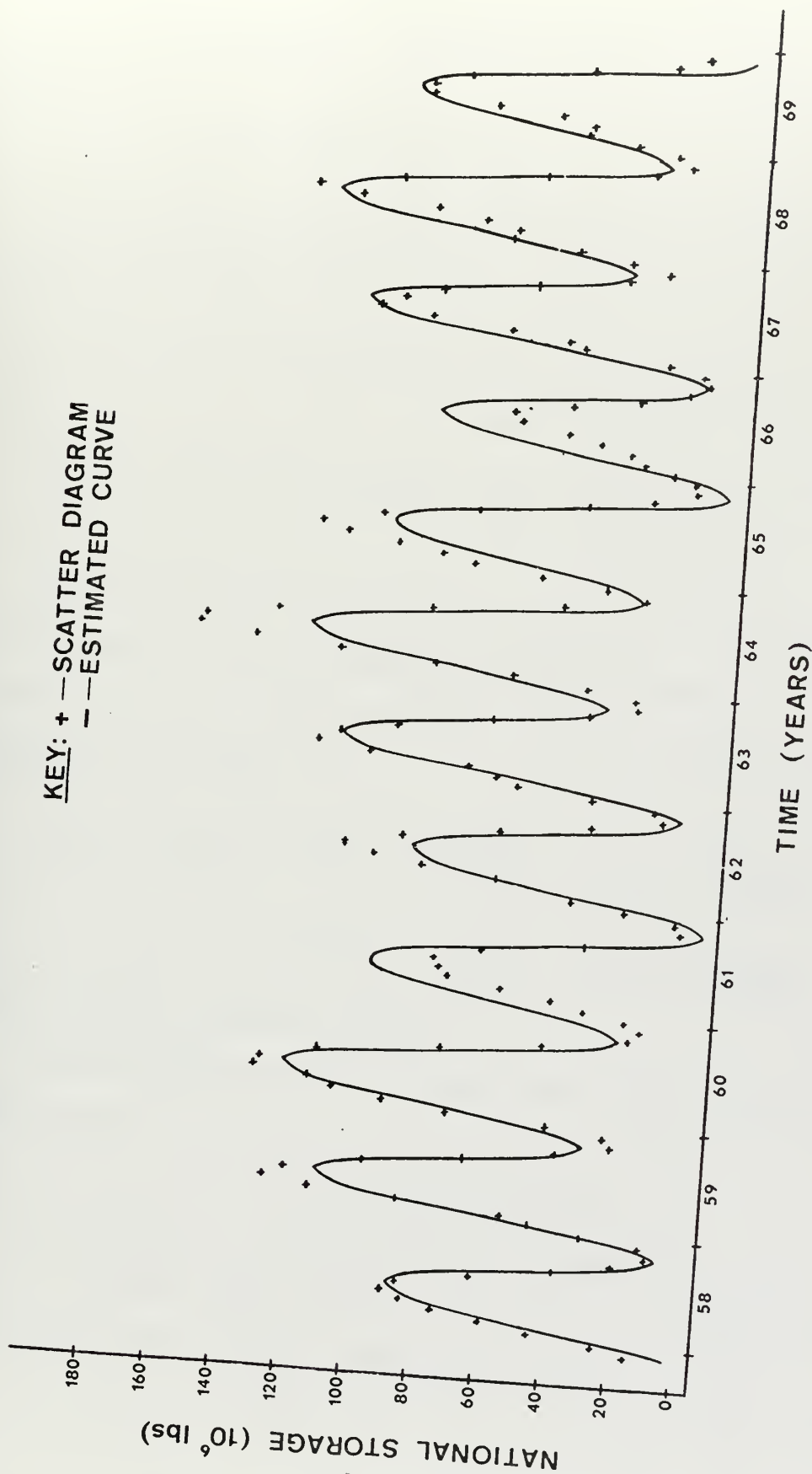


figure 2

where

$$s = [t/12] ; \text{ greatest integer in } t/12$$

and

$$r = 12 (t/12 - [t/12]).$$

Consider the following example for $t=12$, the yearly period, and $t=48$, the four year period:

$t=12 \rightarrow s=[12/12]=1 \rightarrow X(t)=16$; the transformation of the yearly period,

$t=48 \rightarrow s=[48/12]=4 \rightarrow X(t)=64$; the transformation of the four year period.

Estimates for the parameters b_1 , b_2 , b_3 , b_4 , and b_5 were derived using the method of least squares. Placing these estimates in the original relation, the prediction equation was found to be:

$$\begin{aligned} \hat{Y}(t) = & 60620 - 45265 \cos \frac{2\pi X(t)}{16} + 3346 \sin \frac{2\pi X(t)}{16} \\ & - 16440 \cos \frac{2\pi X(t)}{64} - 8019 \sin \frac{2\pi X(t)}{64} \quad (3.1) \end{aligned}$$

and was also plotted in Figure 2.

The estimate for the standard error of residuals, σ was found to be:

$$\hat{\sigma} = 10,298 \times 10^3 \text{ pounds}$$

To determine if the residuals obtained from the least squares solution were serially correlated, the Durbin-Watson d-statistic was computed and found to be:

$$d = .34 .$$

Since a value of $d=2$ would imply zero autocorrelation, the value actually obtained suggested the residuals were autocorrelated. Assuming that this autocorrelation was of the first order, an estimate of the autocorrelation coefficient ρ was determined [Ref. 4], and found to be:

$$\hat{\rho} = .83 .$$

Using this computed value for the estimate of ρ , a second least squares regression was performed on the transformed variables:

$$Y'(t) = Y(t) - \hat{\rho} Y(t-1)$$

$$Z'_i(t) = Z_i(t) - \hat{\rho} Z_i(t-1)$$

where

$$Z_i(t) = \cos \frac{2\pi X(t)}{16} , \text{ etc. } i=2,3,4,5 .$$

The d-statistic resulting from the residuals of the transformed predictive equation was found to be, $d=1.4$. This value of d was less indicative of serial correlation between the residuals of the transformed relationship. Thus it would be better to compute standard errors and confidence intervals for the estimated parameters computed from the transformed variables. The prediction equation in terms of the original variables and the revised estimates of b_1 , b_2 , b_3 , b_4 , and b_5 was found to be:

$$\begin{aligned} \hat{Y}(t) = & 60132 - 44685 \cos \frac{2\pi X(t)}{16} + 3911 \sin \frac{2\pi X(t)}{16} \\ & - 17485 \cos \frac{2\pi X(t)}{64} - 5669 \sin \frac{2\pi X(t)}{64} \quad (3.2) \end{aligned}$$

Again consulting Figure 2, it can be seen that the estimated curve, although failing to identify extreme observations

of peak storage totals, did serve to identify major trends. Thus equation (3.2) could be used to predict the major trends in total pork belly storage and together with basic information pertinent to the market, might prove useful in determining an optimal storage policy for a freezer facility.

V. FUNDAMENTAL MODEL

The objective of this model is to attempt to identify some of the various factors affecting the supply-demand relationships in the pork belly market. Some of the initial assumptions and the limitations of the available data have been previously stated in Section II. The methodology employed in the construction of this model as well as some of the derived relations basically follow those of Hayenga and Hacklander [Ref 1]. Also, since their study served as a useful point of departure for this paper, notation was made as consistent as possible to aid in comparing the two studies.

A. MODEL STRUCTURE

The following five equations illustrate the functional relationships that involve the five endogenous variables (hog slaughter, total supply of pork bellies for in and out-movements, change in pork belly storage, and average monthly price of pork bellies) and predetermined variables selected to describe the system:

$$(1) \quad f_1(Q_h^*, \Delta P_h, INV_{ij}) = u_1$$

$$(2a) \quad f_2(Q_{b_1}^*, Q_h^*) = u_2$$

$$(2b) \quad f_3(Q_{b_2}^*, Q_h^*, \Delta S^*) = u_3$$

$$(3) \quad f_4(\Delta S^*, Q_h^*, P_b^*, S_{t-1}, C_i) = u_4$$

$$(4) \quad f_5(P_b^*, \Delta S^*, Q_{b_j}^*, S_{t-1}, I, R, M) = u_5$$

where * denotes endogenous variables and:

Q_h^* = monthly hog slaughter (in thousands of head per work day; normal work day = 1, Saturdays = .1, and working holidays = .05);

$Q_{b_j}^*$ = monthly pork belly supply, including fresh and frozen stocks (thousands of pounds), where $j=1$ for in-movements, $j=2$ for out-movements;

$\Delta S^* = S_t^* - S_{t-1}$, net change in total pork belly monthly storage (thousands of pounds);

P_b^* = monthly average cash price of 12-14 pound frozen pork bellies (cents per pound);

$\Delta P_h = P_{h,t-1} - P_{h,t-2}$, net change in average monthly hog price lagged one month (dollars per hundred-weight for U.S. 1-2 and 2-3, 200-220 pound barrows and gilts at leading markets);

C_i = the spread between average cash price for 12-14 pound pork bellies at $t-1$ and the five futures' contract average monthly prices at $t-1$, adjusted for costs incident to storage. (See change in storage equation.) The index i is defined as the contract month: Feb=2, Mar=3, May=5, Jul=7, and Aug=8, where $C_i=0$ if $i > t$;

S = monthly total of pork belly storage (thousands of pounds) at time $t-1$;

I = total monthly personal income (billions of dollars) at time $t-1$;

M = monthly rate on new issue, 3-month U.S. Treasury bills;

R = ratio of P_b at $t-1$ to commodity index for food at $t-1$;

INV_{ij} = quarterly pig crop of hogs on farms (thousands of head), where j represents the weight classes: (60-119)=1, (120-179)=2, and (180-219)=3; i represents the report month: Mar=1, Jun=2, Sep=3, and Dec=4;

u = stochastic disturbance term.

1. Supply Equations

It was necessary to derive a relation explaining hog slaughter per month (equation 4.0), since this is the source of the fresh supply for pork bellies for a given month. This equation relates monthly hog slaughter per work day to the change in average hog price lagged one month. Within certain limitations, hog producers might be expected to react to recent hog price changes in determining their marketing intentions. Naturally, this would be a relatively short term effect, as any delay in marketing would increase feed costs and the possibility that a hog would exceed prime weight with a resulting reduction in price received. Likewise, any hog marketed prior to reaching prime weight would also be discounted. An alternative to using the recent change in price as a predetermined variable in the hog slaughter relation, would be to use a predicted change in price [Ref. 1]. However, this would have required additional relations beyond the scope of this paper, encompassing further aspects of the economic forces involved in the hog as well

as cattle market [Ref. 1]. More importantly, the use of a predicted change in hog price would assume that a hog producer could perceive the broad spectrum of economic forces involved in determining future prices of hogs, whereas an extrapolation of recent price changes for future estimates might be a more plausible human tendency.

The quarterly pig crop reports of hogs on farms in the three heaviest weight groups were also used as explanatory variables in this relation. The figures quoted by these reports should be relative measures of the hogs available for market in the month of the report and the two subsequent months. Hogs are usually marketed between 5 to 9 months after farrowing, the average being 6.5 to 7 months. Therefore hogs in the (180-219) weight group would be expected to reach market during the month of a pig crop report. Hogs in the (120-179) and some in the (60-119) weight groups would be marketed in the two months following the report month. Binary dummy variables were used to account for the effects the various weight groups would have on hog slaughter during the months covered by the report.

Because of the variability in total slaughter days for a given month, slaughter per work day was used as a more representative figure with which to make comparisons. It may be seen from daily slaughter figures for a given month [Ref. 6] that Saturday slaughter was approximately 10% of a normal work day and holidays were approximately 5%. Finally, since pork bellies used for bacon predominately come from

hogs under 260 pounds, only barrow and gilt slaughter was considered.

Equation (4.0) is similar to a relation proposed by Hayenga and Hacklander [Ref. 1], the major difference being the use of a recent price change vice a predicted price change for hogs.

As discussed previously, the fresh supply of pork bellies each month was considered to comprise the total pork belly supply only during those months characterized by storage in-movements. Whereas, during the typical months of storage out-movements, the total supply of pork bellies comprised both frozen stocks and the fresh supply.

It appeared that the values for the average weight of pork bellies per hog (Table I) during the months June through September were statistically different than the values for the other eight months of the year. If this were so, the correlation between hog slaughter and fresh pork bellies would not be constant over a year. To test whether the values for α_t were indeed different for the months characterized by out-movements relative to the rest of the year, a multiple comparison test attributed to Scheffé was performed [Ref. 7]. This test was deemed appropriate since the data had been observed prior to the formulation of a hypothesis concerning the data. Concisely stated, the null hypothesis was that the contrast θ between the means of α_t for the four months stated and the remaining eight months was not statistically different from zero at the .05 level of significance. The confidence limits for the contrast θ were found to be:

$$1.143 < \theta < 2.886$$

Since the confidence limits for the contrast θ did not contain the origin, the null hypothesis was rejected.

Although different groupings of α_t will yield other significant contrasts, the choice that was actually used was predicated on the fact that the four months, June through September, also correspond to the typical months of out-of-storage movements of pork bellies. For this reason, two relations were developed to predict total pork belly supply. The first relation (equation 4.1), relates total supply to hog slaughter for the eight months of in-movements. The second relation (equation 4.2), relates total supply to both hog slaughter and net decrease in frozen stocks for the four months of out-movements.

Dummy binary variables for the quarterly report months were added to equation (4.0) to account for the seasonal effects on hog slaughter and act as slope shifters for that predictive equation. Dummy binary variables were also added to equations (4.1) and (4.2) to account for monthly effects on the total pork belly supply.

2. Storage Equation

The storage equation for pork bellies (4.3) relates the monthly net change in storage to the previous month's total storage, the current month's total pork belly supply and average cash price for 12-14 pound pork bellies. It appeared reasonable to assume that storage facility capacity is limited, thus affecting an upper bound on the

total amount of storage possible. Also, each storage facility might be considered to have specified upper and lower limits on storage, outside of which profitability would be proportionately diminished. In addition, the larger the total storage becomes the greater the eventual impact this supply would have when it is finally released. The total pork belly supply of both fresh and frozen stocks might equally affect storage intentions as freezer facilities compete with consumer demand during in-movements and with the fresh supply during out-movements with resulting effects on price. Consequently, average monthly price for pork bellies was also included as an explanatory variable. It was assumed that freezer facility demand for pork bellies is less inelastic than consumer demand for bacon, given a relatively small change in price. Thus storage was thought to be more responsive to the price level for pork bellies.

The spread between average monthly pork belly cash price and each of the five futures' contract average monthly prices was computed (Table III). From these figures, 125 points was subtracted to account for handling costs and an additional 35 points subtracted for each month between the contract month and the current month to reflect storage costs [Ref. 2]. The previous month's values were used as predetermined variables in the storage relation for a given month. These values served as proxies for freezer facility profitability (if a freezer facility bought and stored the equivalent of a contract and at the same time sold a future

to deliver that contract in the future's month, then C_i would be a gross estimate of the profit to be obtained). Spreads were computed starting in November, due to the previously stated requirement that only stocks stored after 1 December are deliverable on contracts the succeeding year. Dummy binary variables were used to account for monthly effects on storage changes.

3. Demand Equation

The pork belly demand equation (4.4) relates price to current change in storage, total supply, and previous month's total storage of pork bellies. One would expect that prices would reflect the total supply, both frozen and anticipated fresh supply, in the face of an expected though unquantified demand. Part of this demand during in-movements, would be change in storage, with the bulk of the demand coming from the consumer and reflected by bacon slicing totals. Monthly personal income and interest rates were also used as explanatory variables in an effort to explain how consumer preferences change with both total income and spendable income. Finally, the ratio of pork belly price to the commodity index for food, lagged one month, was incorporated to explain how pork belly price moved in relation to its previous price relative to other foodstuffs (i.e., the price of pork bellies would be expected to align with the general trend of other commodities). Dummy variables were used to account for monthly effects throughout the year and act as slope shifters in the demand equation.

B. RESULTS

The following equations are the relations previously specified with estimated parameter values properly substituted and obtained by the two-stage least squares method. Below each estimated parameter value is an associated t-statistic in absolute value (ratio of the estimated regression coefficient and the standard error of the regression coefficient). The proportion of variation explained (ratio of the variation attributable to regression and the total variation) is also given.

1. Hog Slaughter Equation

(4.0)

$$\begin{aligned} H^* = & -10.22 + .99\Delta P_h - .006INV_{i1} + .005INV_{i2} \\ & \quad (.59) \quad (.51) \quad (.35) \\ & + .08INV_{i3} + .0005D_1INV_{i1} + .007D_1INV_{i2} \\ & \quad (2.88) \quad (.10) \quad (.81) \\ & - .015D_1INV_{i3} + .01D_2INV_{i1} + .015D_2INV_{i2} \\ & \quad (.92) \quad (3.27) \quad (1.70) \\ & - .048D_2INV_{i3} - 72.79Mar - 101.80Sep - 83.51 Dec \\ & \quad (3.06) \end{aligned}$$

proportion of variation explained = .902.

As might be expected, an increase in price over the previous two months would lead to an increase in slaughter for the present month. Farmers anticipating the continuance of the upward price trend would increase hog deliveries over the month in question. This however is thought to be a relatively short-term effect as might be indicated by the associated t-statistic. As can be seen from the coefficients for

the various weight groups, hogs in the (180-219) group had the largest effect in the month of the report. Hogs in the two lighter weight groups had increasing effects on slaughter in the subsequent two months. These results compare favorably to the results obtained by Hayenga and Hacklander [Ref. 1] for a similar model. However, their use of ΔP_h^* proved to be a more significant variable than ΔP_h used in equation (4.0) by comparison of the associated t-statistics. From this one might conclude that the average hog producer was more cognizant of market forces than was previously credited.

2. Pork Belly Supply (In-Movements) Equation

$$Q_{b1}^* = 14913.31 + 376.60Q_h^* - 8162.14\text{Feb} - 3592.66\text{Mar} \quad (4.1)$$

(5.39)

$$+ 2182.09\text{Apr} + 3396.65\text{May} + 2429.32\text{Oct}$$

$$- 1580.76\text{Nov} + 1112.32\text{Dec}$$

$$\text{proportion of variation explained} = .929 .$$

As may be seen from this equation the effects on pork belly supply contributed by months, increases through the winter, reaching a peak in the spring. This corresponded to the yearly marketing patterns for hogs, from which pork bellies are derived. The effects contributed by the various months also partially account for the variability in the weight of pork bellies per hog (α_t), that was cited previously.

3. Pork Belly Supply (Out-Movements) Equation

$$Q_{b2}^* = 563.25 + \underset{(2.50)}{372.28Q_h^*} - \underset{(2.82)}{.82\Delta S^*} + 9622.34\text{Jun} \quad (4.2)$$

$$+ 6274.70\text{Jul} + 7579.07\text{Aug}$$

proportion of variation explained = .855.

The change in storage (ΔS^*) is a negative quantity and thus adds to the total pork belly supply during out-movements. The smaller coefficient of hog slaughter (Q_h^*) in equation (4.2) compared to equation (4.1), reflected the decrease in the pounds of pork bellies per hog during the summer months. This coefficient is approximately equivalent to α_t times the number of slaughter work days during month t . The effects produced on total supply by the three months relative to September, also partially account for the variability of α_t . Equations (4.1) and (4.2) were a direct result of the basic assumptions described by equations (1.0) and (1.1). The associated power of predictability of the above equations would therefore be governed by the validity of the basic assumptions.

4. Pork Belly Change in Storage Equation

$$\Delta S^* = -75539.63 - \underset{(.63)}{.133S} + \underset{(1.04)}{.245Q_{bj}^*} - \underset{(.163)}{69.61P_b^*} \quad (4.3)$$

$$+ \underset{(1.20)}{30462.12C_2} + \underset{(.47)}{3611.28C_3} - \underset{(.83)}{5594.07C_5}$$

$$+ \underset{(1.56)}{7781.13C_7} - \underset{(1.23)}{3701.35C_8} + 19978.54\text{Feb}$$

+ 69680.88Mar + 80103.13Apr + 65367.51May
 + 43886.74Jun + 24749.81Jul + 19355.38Aug
 + 42342.38Sep + 57205.22Oct + 64591.73Nov
 - 11006.86Dec

where

j = 1; for in-movements
 2; for out-movements

proportion of variation explained = .95 .

Pork belly storage (S) had a negative effect on the current monthly change in storage. This was thought to be due to the fact that adding to an already large existing supply would serve to increase risk to the freezer facilities. Since the time of typical out-movements was only one-third of an entire storage season, stocks would have to be depleted in half the time taken for accumulation. Thus a relatively large total storage supply in May, for example, would have to be met by a corresponding large increase in consumer demand or shortage of fresh supply during the summer to insure any sort of price stability. This fact coupled with the observed tendency for spread to decrease substantially with the approach of summer (Table III) would impinge on freezer facility profitability were they to arbitrarily increase storage stocks. Likewise, during out-movements, a large total storage would contribute to a greater change in storage due to the relatively short period in which to reduce inventory. The additional prospect of an increase in fresh supplies in the fall facing a decrease in consumer demand

would remove any advantage to be gained by maintaining storage supplies through the summer. Net storage changes varied from 10% to 22% of the available fresh supply during in-movements [Ref. 6], or equivalently, consumer demand accounted for 78% to 90% of the fresh supply during this period. From this it was concluded that actions by freezer facilities, being the less dominant force in the market, would have less of an effect on price. Thus during periods of large fresh supply, they would store more pork bellies as a result of the probable downward shift in the cash price. During periods of out-movements, frozen stocks would be augmenting the fresh supply and if this were large relative to demand, would have an increasing negative effect on storage changes. The coefficient of pork belly price (P_b^*) suggested that the level of price had a negative effect on storage additions and a positive effect on storage depletions. One possible explanation for this could be that as the price level of pork bellies increased, the margin of expected profit from storing the product would decrease. Although there is no absolute ceiling to which prices can rise, historically the range has been approximately defined. As prices approach the upper limits of this range, the probability of a continued rise on which storage profits hinge, would be greatly reduced.

Table III lists the spread between current cash price and futures' price for each contract month, uncorrected for storage and incidental costs. As may be seen from this

table, the spread for a given month after correcting for storage and handling costs, generally decreased over the contract months. This would indicate that hedging would be less and less profitable for the later contract months. However, hedging did remain profitable for the most part, up through May for most contracts. The positive coefficients of C_2 , C_3 , and C_7 indicated this favorable spread, whereas the negative coefficients of C_5 and C_8 indicated the reduced profitability in hedging for those contract months.

5. Pork Belly Demand Equation

(4.4)

$$\begin{aligned}
 P_b^* = & 51.33 - .00004\Delta S^* - .00016Q_b^* - .00017S \\
 & \quad (.402) \quad (1.60) \quad (1.71) \\
 & - .046I + .755R + .373M - 1.74\text{Feb} + 2.38\text{Mar} \\
 & \quad (.804) \quad (3.18) \quad (1.53) \\
 & + 7.14\text{Apr} + 10.30\text{May} + 7.12\text{Jun} + 4.08\text{Jul} \\
 & - .58\text{Aug} - 5.05\text{Sep} - 7.15\text{Oct} - 5.15\text{Nov} - 1.15\text{Dec} \\
 & \text{proportion of variation explained} = .88.
 \end{aligned}$$

As previously stated, demand for storage stocks is small compared to consumer demand and therefore the negative coefficient for ΔS^* was not surprising. Change in storage might be viewed as an effect and price the cause. A large anticipated supply served to lower the expected price for a given month. A large overhanging supply would have a negative effect on price as indicated by the negative coefficient for total storage. Normally, if bacon is to be considered a basic foodstuff, as personal income increased a

lesser proportion would be spent on bacon. The negative coefficient for income would indicate this. However, the positive effect indicated by interest rates (which were rising during the period of observations) might be interpreted as reflecting an increase in discretionary income as consumers avoided purchases involving long-term costs (i.e., homes, automobiles, appliances, etc.). Thus more of the consumer dollar would be freed for the purchases of foodstuffs such as bacon. These two results appeared to be conflicting and a more intensive analysis may be needed to reveal the true economic interpretation. In light of the relative values of the associated t-statistics, the latter interpretation was considered the more meaningful. The positive coefficient for (R) was interpreted to mean that as the commodity index rose (which was indicative of this period), pork belly prices adjusted upwardly in line with the prices of other commodities. Prices rose through the in-movement portion of the storage period and fell through the out-movement portion as indicated by the monthly effects. This result appeared reasonable, in that prices might rise in anticipation of a relative increase in demand and fall as that demand is realized.

VI. CONCLUSION

Cogito ergo sum.

René Descartes

The results obtained by the two technical models were considered quite good in light of the basic techniques employed. The adaptability of their respective predictive powers in determining a storage policy would hinge on the ability to incorporate other market information not given by these models. Determining total national storage at any one time would not be an end in itself but would have to be used in conjunction with associated price moves. The correlation between price and supply was not as strong as might be desired and consequently predicting storage changes might not be a powerful tool in determining freezer facility policies.

Various pertinent economic forces were identified in the fundamental model. Some of the explanatory variables used were not as significant in determining the structure of the model as were anticipated. This was evidenced by the associated t-statistics of these terms. The proportion of variation explained in each relation was generally quite high and was considered satisfactory. Obviously there remain functional relationships between other explanatory variables yet to be determined. Two of these variables were thought to be hog price and hog-corn price ratio lagged by two to three years, as suggested by Wold [Ref. 8], for use

in analyzing the motivating factors in hog production. For various reasons hog producers cannot alter their marketing intentions at will, but slowly adjust to the prevalent market forces such as feed costs and hog prices. This naturally has a corresponding effect on the supply of pork bellies and in turn, storage policies.

As was stated earlier in this paper, the methodology used for the fundamental model was derived from a similar study conducted by Hayenga and Hacklander [Ref. 1]. Equation (4.0) was the only relation subject to direct comparison with a similar equation in their study. The results obtained were considered essentially equivalent in view of the different data series used in each paper. Their study covered the period April, 1963 to June, 1968 and the corresponding hog supply equation used resulted in 84% proportion of variation explained.

It was initially desired to determine a storage policy based on the results of these models. However, certain decision rules would have to be formulated along with a freezer facility's stated constraints. In addition, a probability statement concerning prices for an entire storage season would have to be constructed. This last requirement was beyond the capability of the models proposed in this paper. If the above information could be derived, an optimal storage policy might be proposed using dynamic programming techniques. An alternative solution would be to attempt to maximize freezer facility profit a month at a time. This approach would in all probability lead to a sub-optimal

policy over the long term and was therefore rejected. The problem of a yearly storage policy remains subject for further investigation and more advanced modeling techniques.

		Years		
		67	68	69
Months	α_{ij}			
	1	19.040	19.319	18.878
	2	19.902	19.621	18.937
	3	19.638	19.521	19.179
	4	19.572	20.633	19.914
	5	19.816	21.230	20.115
	6	19.241	18.368	18.982
	7	17.671	16.593	17.436
	8	16.935	17.685	17.070
	9	17.486	17.527	17.545
	10	19.267	18.109	19.237
	11	20.509	20.330	19.520
12	20.376	20.313	20.445	

TABLE I. Average weight of pork bellies per hog per month for years 1967, 1968, and 1969.

Source	SS	D.F.	MS	T.S.
Common Slope	$\sum_{ij} (\beta - \beta_{ij})^2 \sum_k (x_{ijk} - \bar{x}_{ij})^2$	1		
Slope	$\sum_{ij} (\hat{\beta}_{ij} - \hat{\beta})^2 \sum_k (x_{ijk} - \bar{x}_{ij})^2$	5	$\frac{SS_{SLOPE}}{5}$	$\frac{MS_{SLOPE}}{MS_{WITHIN}}$
Cell Mean	$\sum_{ijk} (\bar{y}_{ij} - \mu_{ij})^2$	6		
Residuals	$\sum_{ijk} [y_{ijk} - \bar{y}_{ij} - \hat{\beta}_{ij} (x_{ijk} - \bar{x}_{ij})]^2$	N-12	$\frac{SS_{WITHIN}}{N-12}$	
Total	$\sum_{ijk} [y_{ijk} - \mu_{ij} - \beta_{ij} (x_{ijk} - \bar{x}_{ij})]^2$	N		

TABLE II. Analysis of covariance operational table.

Futures' Contract Months

	Feb	Mar	May	Jul	Aug
Mar			2.43	2.73	2.07
Apr			2.37	2.89	2.51
67 May				1.68	1.26
Jun				1.34	.05
Jul					-.35
Nov	4.55	4.33	4.41	4.57	3.72
Dec	3.24	3.13	3.32	3.57	2.89
Jan	2.84	1.97	2.20	2.42	1.58
Feb		1.79	1.78	1.81	1.02
68 Mar			2.94	3.17	2.28
Apr			2.76	3.10	2.37
May				2.04	1.54
Jun				.06	0
Jul					-.06
Nov	4.27	4.27	4.52	4.94	4.37
Dec	3.12	3.06	3.20	3.39	2.48
Jan	2.58	2.32	2.59	2.80	1.93
Feb		2.87	3.41	3.70	2.84
69 Mar			5.12	5.58	4.66
Apr			2.68	3.26	2.38
May				2.68	1.93
Jun				-.51	-1.02
Jul					-2.78

TABLE III. Spread between cash and futures' prices from March 1968 through July 1969.

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13. ABSTRACT

Various models relating to the storage of pork bellies are studied. An analysis of covariance is used to determine if specific regional weekly storage totals can be used to predict national monthly storage totals. A technical forecast model in the form of a Fourier series is presented, derived from past national storage data. Finally an econometric model relating pork belly demand, change in storage supply, hog slaughter, and total pork belly supply is formulated and fitted to the data base.

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